

# Analytical representation of experimental light scattering phase functions measured in seas, oceans and lake Baykal

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**Abstract** — This paper analyzes 20 recently released phase functions of light scattering by natural water (18 marine and 2 lake) together with 15 Petzold marine phase functions. All phase functions with regression coefficients exceeding 0.998 are represented in an analytical form that allows to compute their values at any scattering angle and the values of any derived parameter. The values of statistics with accompanied inherent optical properties that are necessary to compute modulation transfer functions are presented in a tabular form.

## INTRODUCTION

In many practical cases of radiative transfer modeling it is very important to represent results of measurements of light scattering phase functions in a form of analytical equation. To obtain a meaningful analytical expression for seawater phase functions is very tricky task. Because of a high anisotropy, these phase functions and their logarithms are poorly represented by polynomials of any reasonable order. This is a consequence of the fact that Legendre polynomial decomposition of these phase functions contains several thousands of meaningful terms.

This work presents results of successful representation of new 20 phase functions measured by Mankovsky [1] and 15 phase functions published by Petzold [2] in a form of analytical expression that contains only seven parameters and represents the original numerical tables with the regression coefficients larger than 0.998 for Mankovsky phase functions and larger than 0.999 for Petzold phase functions. Additional parameters, which are important for computations of image transfer in seawater, are also computed and presented in the form of regression that connect them with other inherent optical properties.

## ANALYTICAL REPRESENTATION

Due to an extreme anisotropy of seawater scattering phase functions, numerous earlier attempts to represent marine phase functions of light scattering in analytical form either fail or produce poor results. The first successful attempt was proposed in Ref. [3] and our analysis is an extension and improvement of the method proposed in this paper. The analysis consists of customized fitting with minimization of regression coefficient. It was found that a polynomial

regression of phase function logarithm against square root of scattering angle  $\theta$  produces the best results with regression coefficients distributed around value of 0.999.

$$p(\theta) = \exp\left(\sum_{n=0}^6 c_n (\sqrt{\theta})^n\right), \quad r^2 \sim 0.999. \quad (1)$$

The phase function in Eq. (1) is normalized according to the convenience accepted in marine optics:

$$0.5 \int_0^\pi p(\theta) \sin \theta d\theta = 1. \quad (2)$$

The seven regression coefficients ( $c_0 \div c_6$ ) for each of 20 Mankovsky and 15 Petzold phase functions of scattering are presented, correspondingly, in Tables 1 and 2. Two samples of comparison of experimental values of  $p(\theta)$  with the values obtained with Eq. (1) are shown in Figures 1 and 2.

Equation (1) allows us to compute all important properties of waters associated with these phase functions.

Table 1. Coefficients to Eq. (1) for 20 numbered Mankovsky phase functions [1]; the numbering here starts from the top.

$c_0$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$
12.469	-45.099	62.778	-37.338	-13.128	22.983	-6.3979
10.799	-32.012	37.607	-14.917	-19.179	20.440	-5.1816
10.857	-31.494	34.853	-16.519	-9.3988	12.653	-3.3721
14.233	-66.274	112.19	-86.477	12.345	16.031	-5.6094
13.276	-55.438	90.605	-67.294	5.6820	15.804	-5.1711
8.3022	-11.706	-15.955	54.837	-67.644	37.508	-7.5588
7.9010	-8.5579	-24.545	65.429	-73.632	39.087	-7.7336
10.348	-23.204	-6.3717	70.489	-96.181	53.038	-10.455
12.094	-41.132	50.489	-24.352	-9.3636	12.934	-3.1985
14.051	-70.099	165.45	-226.06	166.75	-62.170	9.2855
10.425	-26.698	10.652	43.957	-77.833	47.219	-9.7692
11.953	-44.577	83.727	-100.57	63.020	-17.898	1.6602
11.456	-35.700	43.529	-33.044	13.188	-1.9456	-0.01313
11.220	-35.314	49.886	-48.025	24.508	-4.9875	0.13011
13.446	-60.498	125.68	-154.53	103.69	-34.929	4.6379
13.254	-55.874	99.135	-97.451	47.060	-8.7793	0.10543
10.045	-21.637	-4.4404	52.645	-66.715	34.590	-6.4741
11.646	-38.767	52.491	-37.966	7.7029	4.5656	-1.7821
8.7946	-6.5846	-57.663	131.96	-124.76	55.332	-9.3715
10.742	-27.380	8.0476	41.470	-62.707	34.333	-6.5317



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## APPENDIX: A FORTRAN FUNCTION TO COMPUTE MANKOVSKY AND PETZOLD PHASE FUNCTIONS OF LIGHT SCATTERING FOR ARBITRARY ANGLE

```
C **** real function mplnphf(p,N,angle)
C **** Computes natural logarithm of Mankovsky or Petzold phase
C function. if p='m' or 'M' this function computes Mankovsky
C phase function, if p='p' or 'P' this function computes Petzold
C phase function. 1≤N≤15 for Petzold and 1≤N≤20 for Mankovsky.
C angle is a scattering angle in degrees.
C **** implicit none
integer i, N, Nm
real angle, x, thet, c(0:6,35)
character p
thet = angle*ATAN(1.0)/45.0
if (p .eq. 'm' .or. p .eq. 'M') then
    Nm = N+15
    if (Nm .gt. 35) Nm = 35
else if (p .eq. 'p' .or. p .eq. 'P') then
    Nm = N
    if (Nm .gt. 15) Nm = 15
else
    Nm = 0
end if
if (Nm .ne. 0) then
    call initc(c)
    x = SQRT(thet)
    mplnphf = c(0,Nm)+x*(c(1,Nm)+x*(c(2,Nm)+x*
        & (c(3,Nm)+x*(c(4,Nm)+x*(c(5,Nm)+x*c(6,Nm))))))
end if

return
end

C **** subroutine initc(c)
C **** implicit none
integer i, j, k
real c(0:6,35)

open(11, file='c.in',status='old')
read(11,*)
do j = 1,15
    read(11,*) (c(i,j), i=0,6)
end do
read(11,*)
do j = 1,20
    k = j+15
    read(11,*) (c(i,k), i=0,6)
end do
close(11)

return
end
C ****
```

file 'c.in':						
Petzold c(0:6,1:15):						
12.29463	-55.043	134.23	-193.23	149.02	-57.929	9.0317
11.32499	-45.768	104.56	-145.18	106.71	-38.639	5.5094
11.66488	-48.635	111.97	-153.75	111.37	-39.755	5.5927
12.29880	-53.032	126.42	-185.22	146.91	-59.155	9.6300
12.06609	-50.705	118.95	-172.19	134.04	-52.659	8.3485
12.17760	-50.155	110.20	-151.59	115.70	-45.639	7.3571
12.05656	-48.072	100.25	-131.05	94.482	-34.872	5.2289
11.74086	-46.257	100.16	-138.28	105.08	-40.762	6.3982
9.38649	-32.619	77.634	-128.15	113.01	-48.471	8.0665
9.34029	-24.362	34.734	-40.67	25.605	-7.274	0.74276
9.44031	-25.709	41.649	-56.117	41.540	-14.874	2.1002
11.25595	-36.955	54.016	-44.47	13.073	2.3826	-1.3574
11.98498	-49.121	108.22	-145.70	105.87	-38.904	5.7801
11.95112	-49.667	111.72	-151.09	108.03	-37.991	5.2398
9.07291	-41.835	134.13	-243.06	221.35	-97.049	16.4480
Mankovsky c(0:6,16:20):						
12.469	-45.099	62.778	-37.338	-13.128	22.983	-6.3979
10.799	-32.012	37.607	-14.917	-19.179	20.440	-5.1816
10.857	-31.494	34.853	-16.519	-9.3988	12.653	-3.3721
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11.953	-44.577	83.727	-100.57	63.020	-17.898	1.6602
11.456	-35.700	43.529	-33.044	13.188	-1.9456	-0.013128
11.220	-35.314	49.886	-48.025	24.508	-4.9875	0.13011
13.446	-60.498	125.68	-154.53	103.69	-34.929	4.6379
13.254	-55.874	99.135	-97.451	47.060	-8.7793	0.10543
10.045	-21.637	-4.4404	52.645	-66.715	34.590	-6.4741
11.646	-38.767	52.491	-37.966	7.7029	4.5656	-1.7821
8.7946	-6.5846	-57.663	131.96	-124.76	55.332	-9.3715
10.742	-27.380	8.0476	41.470	-62.707	34.333	-6.5317
12.558	-47.271	77.913	-82.443	48.195	-13.513	1.3736

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